

Soft gluons are heavy and rowdy *

Reinhard Alkofer¹, Pedro Bicudo², Stephen R. Cotanch³,
Christian S. Fischer⁴ and Felipe J. Llanes-Estrada⁵.

¹ Inst. für Physik, Karl-Franzens Univ. 1-8010 Graz, Austria.

² Instituto Superior Tecnico, Av. Rovisco Pais, 1049-001 Lisboa, Portugal.

³ Physics Department, North Carolina State University, Raleigh 27695 NC USA.

⁴ Gesellschaft für Schwerionenforschung, Darmstadt, Germany.

⁵ Fisica Teorica I, Universidad Complutense de Madrid, 28040 Madrid Spain,
E-mail: flanes@fis.ucm.es

We study dynamical mass generation in pure Yang-Mills theory and report on a recently developed ansatz that exactly solves the tower of Dyson-Schwinger equations in Landau gauge at low Euclidean momentum, featuring enhanced gluon-gluon vertices, a finite ghost-gluon vertex in agreement with an old argument of Taylor, and an IR suppressed gluon propagator. This ansatz reinforces arguments in favor of the concept of a gluon mass gap at low momentum (although the minimum of the gluon's dispersion relation is not at zero momentum). As an application, we have computed the spectrum of oddballs, three-gluon glueballs with negative parity and C-parity. The three body problem is variationally solved employing the color density-density interaction of Coulomb gauge QCD with a static Cornell potential. Like their even glueball counterparts, oddballs fall on Regge trajectories with similar slope to the pomeron. However their intercept at $t = 0$ is smaller than the ω Regge trajectory and therefore the odderon may only be visible in experimental searches (for example at BNL) with higher $-t$ than conducted to date at DESY.

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In a free Yang-Mills theory gauge boson mass terms break gauge invariance and cannot be part of the Lagrangian. Moreover, if self-interactions are incorporated perturbatively, to any finite order in perturbation theory the radiative corrections do not generate mass terms. However non-perturbative effects can potentially provide dynamical suppression of the gauge boson propagator via a dressing function

$$\frac{1}{p^2} \rightarrow \frac{Z(p^2)}{p^2} . \quad (1)$$

If this function Z is proportional to p^2 in the limit $p \rightarrow 0$ then the complete propagator behaves as that of a Yukawa exchange of finite mass. This behavior was conjectured by Cornwall [1]. Dynamical suppression in gluodynamics, unlike the electroweak Lagrangian, is not facilitated by coupling to an external Anderson-Higgs field, but can only be an intrinsic phenomenon due to the non-linearity and strength of the self-coupling.

The fact that the positions of resonances in the hadron spectrum seem to follow a discrete pattern is a clear indication that the gluon degrees of freedom entail a mass gap of order 800 MeV.

This has been previously incorporated in phenomenological models using a constituent gluon mass [2, 3], a waveguide cut-off frequency such as the bag model [4], or a quantum string excitation in the flux tube model [5].

In the last years the suppression of the gluon propagator has been put on firm theoretical footing in the framework of the Yang-Mills Dyson-Schwinger equations [6, 7, 8, 9], and has also received support from numerical lattice calculations [10, 11, 12].

The studies in [6, 8, 9], employing sophisticated ansätze for the interaction vertices were able to provide analytical and numerical solutions for the infrared behavior of the Yang-Mills ghost and gluon propagators. The novelty was that the dressing function for the gluon propagator in Landau gauge was more suppressed than p^2 , leading to an infinite gluon mass at zero momentum. Conversely, the dressing function for the ghost propagator turned out to be infrared singular instead of suppressed. In terms of a real constant $\kappa \in (0.5, 0.6)$ (the calculation of [9] yields 0.596), they read

$$\lim_{p \rightarrow 0} Z(p^2) = \text{ct} \cdot (p^2)^{2\kappa} \quad \lim_{p \rightarrow 0} G(p^2) = \text{ct} \cdot (p^2)^{-\kappa} . \quad (2)$$

In [7] further progress has been made by observing that one can make a self-consistent ansatz that simultaneously solves all of the Yang-Mills wave equations in the far infrared. The result for the dressing of the Green's function with m gluon legs and $2n$ ghost legs,

$A[m, 2n]$, is

$$\lim_{\text{all } p \rightarrow 0} A[m, 2n](p^2) = \text{ct} \cdot (p^2)^{(n-m)\kappa} . \quad (3)$$

By “dressing” we understand that the canonical dimension of the Green’s function is still multiplying the result, and by “all” that all the momenta entering the scattering amplitude need to tend to zero simultaneously. A proof employing the skeleton expansion has been given in [7] and [8]. In addition to the aforementioned gluon suppression and ghost enhancement, one finds from eq. (3) that the ghost-gluon vertex ($n = 1, m = 1$) is IR finite in agreement with an argument of Taylor [13] and that the three and four-gluon vertices are enhanced by $(p^2)^{-3\kappa}$ and $(p^2)^{-4\kappa}$ dressings respectively. These relations between the dressings force cancellations such that all the effective charges defined from the primitively divergent vertices in the Lagrangian are finite in the IR, and α_s presents an IR fixed point, which theoretically supports approaches based on conformal theory [14] and is compatible with analytical perturbation theory [15, 16].

The demonstration that eq. (3) indeed solves the tower of DSE’s has been given in terms of the skeleton expansion. Here we give an argument that is independent of this expansion, sidestepping convergence issues.

We will examine in turn all the contributions to the RHS of the Dyson Schwinger equation for $A[m, 2n]$, assume eq. (3) for the coupled-channel Green’s functions appearing there, and show that the leading IR divergence yields again our counting rule eq. (3).

There are several formulations of the Dyson-Schwinger equations, we employ a very intuitive one based on a complete resummation of the perturbative series (without the intermediate skeleton expansion), that can conceivably be generalized by analytical continuation, and one should remember that the DSE’s can be directly formulated as identities in the path-integral formalism.

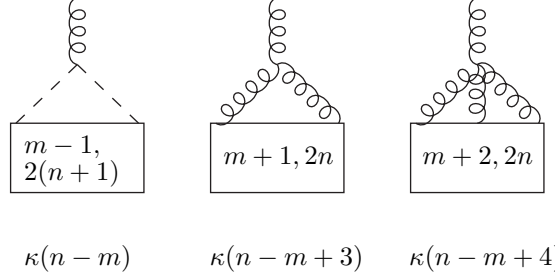
We will draw a box for the Green’s function with m gluon and n ghost legs labeled by the indices $m, 2n$. Its DSE reads

(4)

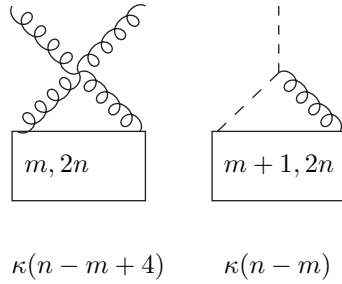
$$\boxed{m, 2n} = \text{Tree} + (2a) + (2b) + (3)\delta(m) .$$

The tree-level diagrams are the lowest order in perturbation theory and have their canonical dimension. All others involve loop integrals and can feature an anomalous infrared exponent. Diagrams of type (2a) have at least one external gluon leg coupled to a vertex that is not attached to any other external leg. These diagrams can be resummed

to all orders to one of the three following classes (we write below the corresponding infrared exponents). All propagators are understood as fully dressed and carrying their dimensions given by eq. (2).



The ghost-triangle type diagram on the left is the IR-dominant and yields the behavior of the $A[m, 2n]$ function in eq. (4). Diagrams marked (2b) (see sketch below, left diagram) arise due to the four-gluon coupling, that allows two external gluon legs linked together. Finally we have diagrams of type (3), that we define as those with no external gluon legs ($m = 0$) that require separate treatment. These are the only loop diagrams in the Dyson-Schwinger equations for multighost scattering kernels. Since there is only one bare ghost-gluon vertex in the Lagrangian these can all be resummed to the diagram on the right (that is leading and again yields our IR counting):



This exhausts the possible types of proper diagrams that can be resummed from the perturbative Yang-Mills series.

Once we have established that the ansatz eq. (3) is a valid solution of the Dyson-Schwinger equations in Landau gauge, we turn to phenomenological applications. These are based in the sister Coulomb gauge, also transverse but with a Hamiltonian formulated

at equal time. The obvious phenomenological consequence of the gluon mass gap is that hybrid mesons ($q\bar{q}g$), glueballs, and other states whose leading wavefunction in Fock space involve constituent glue, will be rather massive, in agreement with estimates in lattice gauge theory [17, 18]. We complement lattice studies in that our semianalytical methods in the continuum can readily be used to study higher spin states, not easily accessible in the lattice. Since glueballs do not have exotic quantum numbers they are the most difficult unconventional states to be disentangled from the meson spectrum. However they leave some trace in Regge phenomenology. There is no doubt among theorists that Regge behavior is present in QCD, but the BFKL, BKP equations are valid only for $s \gg -t \gg \Lambda_{QCD}$, whereas the experimental evidence so far has been gathered at small or zero $-t$ (mostly in the form of total cross sections), therefore non-perturbative approaches are necessary to study Regge trajectories at low t . The time-honored approach to obtain the intercept of a Regge trajectory at $t = M^2 = 0$ has been to study the resonances on the positive (M^2, J) quadrant of a Chew-Frautschi plot.

We address these states by means of a model Hamiltonian employing the Coulomb gauge formulation and fields, where the kernel is approximated by a charge-density to charge-density interaction with a Cornell potential. In a first approximation one neglects (suppressed) transverse gluon exchange, that can later be incorporated in perturbation theory (a model computation along these lines is available [19]). In the gluon sector this effective QCD Hamiltonian is

$$\begin{aligned} H_{eff}^g &= Tr \int d\mathbf{x} [\boldsymbol{\Pi}^a(\mathbf{x}) \cdot \boldsymbol{\Pi}^a(\mathbf{x}) + \mathbf{B}_A^a(\mathbf{x}) \cdot \mathbf{B}_A^a(\mathbf{x})] \\ &- \frac{1}{2} \int d\mathbf{x} d\mathbf{y} \rho_g^a(\mathbf{x}) V(\mathbf{x}, \mathbf{y}) \rho_g^a(\mathbf{y}) , \end{aligned} \quad (5)$$

with color charge density $\rho_g^a(\mathbf{x}) = f^{abc} \mathbf{A}^b(\mathbf{x}) \cdot \boldsymbol{\Pi}^c(\mathbf{x})$, gauge fields \mathbf{A}^a , conjugate momenta $\boldsymbol{\Pi}^a = -\mathbf{E}^a$ and Abelian components $\mathbf{B}_A^a = \boldsymbol{\nabla} \times \mathbf{A}^a$, for $a = 1, 2, \dots, 8$. The Fourier transform to momentum space yields

$$\mathbf{A}^a(\mathbf{x}) = \int \frac{d\mathbf{q}}{(2\pi)^3} \frac{1}{\sqrt{2\omega_k}} [\mathbf{a}^a(\mathbf{q}) + \mathbf{a}^{a\dagger}(-\mathbf{q})] e^{i\mathbf{q} \cdot \mathbf{x}} , \quad (6)$$

$$\boldsymbol{\Pi}^a(\mathbf{x}) = -i \int \frac{d\mathbf{q}}{(2\pi)^3} \sqrt{\frac{\omega_k}{2}} [\mathbf{a}^a(\mathbf{q}) - \mathbf{a}^{a\dagger}(-\mathbf{q})] e^{i\mathbf{q} \cdot \mathbf{x}} , \quad (7)$$

satisfying the Coulomb gauge transverse condition,

$$\mathbf{q} \cdot \mathbf{a}^a(\mathbf{q}) =$$

$(-1)^\mu q_\mu a_{-\mu}^a(\mathbf{q}) = 0$. Here $a_\mu^a(\mathbf{q})$ ($\mu = 0, \pm 1$) are the bare gluon Fock operators from

which, by a Bogoliubov-Valatin canonical transformation, the dressed gluon or quasiparticle operators, $\alpha_\mu^a(\mathbf{q}) = \cosh \Theta(q) a_\mu^a(\mathbf{q}) + \sinh \Theta(q) a_\mu^{a\dagger}(-\mathbf{q})$, emerge. This similarity transformation is a hyperbolic rotation similar to the BCS fermion treatment. These operators excite constituent gluon quasiparticles from the BCS vacuum, $|\Omega\rangle_{\text{BCS}}$, and satisfy the transverse commutation relations, $[\alpha_\mu^a(\mathbf{q}), \alpha_\nu^{b\dagger}(\mathbf{q}')] = \delta_{ab}(2\pi)^3 \delta^3(\mathbf{q} - \mathbf{q}') D_{\mu\nu}(\mathbf{q})$, with $D_{\mu\nu}(\mathbf{q}) = \left(\delta_{\mu\nu} - (-1)^\mu \frac{q_\mu q_\nu}{q^2} \right)$. Finally, the quasiparticle or gluon self-energy, $\omega(q) = qe^{-2\Theta(q)}$, satisfies a gap equation [20]. A more sophisticated treatment is also available in the literature [21],

In this report we concentrate on glueballs (hybrid mesons can be found in [23]). Two-gluon glueballs [24] have been variationally computed in the Russell-Saunders (or $L-S$) scheme. The ground state is the $L=0, S=0$ scalar $J^{PC} = 0^{++}$ that we employ to fix the cutoff regulating the divergence in the gluon self-energy, to agree with the calculation of Morningstar and Peardon [17]. The rest of the parameters being fixed from the quark sector, all other glueball masses are predictions of the approach. If we plot the spectrum in a Chew-Frautschi plot to reveal Regge trajectories, this state cannot be on the leading (left-most) trajectory since it has total $J=0$. Its orbital excitations with $S=0, L=J=2, 4, \dots$ fall on a linear Regge trajectory with slope close to the pomeron. The second state is the 2^{++} in which both spins are aligned, and this one does fall on the pomeron Regge trajectory as observed first by Simonov ([2] and references therein). This is natural since the intercept of a Regge trajectory with a line of integer J usually (but not always) produces a resonance in the spectrum. However, the pomeron having a small slope (0.2-0.3 by modern fits [22]) cannot be represented by ordinary mesons. Casimir color scaling between the gluon-gluon and the quark-antiquark interaction provide for an enhanced string tension in the glueball system that supports the pomeron-glueball conjecture (the slope $\alpha'(t) \propto 1/\sigma$). Turning now to the three body problem, the three-gluon variational wavefunction is

$$|\Psi^{JPC}\rangle = \int d\mathbf{q}_1 d\mathbf{q}_2 d\mathbf{q}_3 \delta(\mathbf{q}_1 + \mathbf{q}_2 + \mathbf{q}_3) \quad (8)$$

$$F_{\mu_1\mu_2\mu_3}^{JPC}(\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3) C^{abc} \alpha_{\mu_1}^{a\dagger}(\mathbf{q}_1) \alpha_{\mu_2}^{b\dagger}(\mathbf{q}_2) \alpha_{\mu_3}^{c\dagger}(\mathbf{q}_3) |\Omega\rangle_{\text{BCS}} ,$$

with summation over repeated indices. The color tensor C^{abc} is either totally antisymmetric f^{abc} (for $C=1$) or symmetric d^{abc} (for $C=-1$). Boson statistics thus requires the

$C = -1$ oddballs to have a symmetric space-spin wavefunction taken here to have form

$$F_{\mu_1\mu_2\mu_3}^{JPC}(\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3) = [c_{12}f(q_1, q_2) + c_{23}f(q_2, q_3) + c_{13}f(q_1, q_3)][Y_L^\lambda(\hat{\mathbf{q}}_1) + Y_L^\lambda(\hat{\mathbf{q}}_2) + Y_L^\lambda(\hat{\mathbf{q}}_3)] , \quad (9)$$

$$c_{12} = \langle 1\mu_1 1\mu_2 | s\mu_s \rangle \langle s\mu_s 1\mu_3 | S\mu \rangle \langle L\lambda S\mu | JM \rangle , \quad (10)$$

sufficient to analyze the lightest states. The other two coefficients in Eq. (9) can be obtained by permuting the indices in Eq. (10). Several forms for the variational radial wavefunction, $f(q, q')$, involving two variational parameters, β and β' , were investigated including a separable form, $f(q)f(q')$, which only involves one parameter. From previous experience [23], reliable, accurate variational solutions can be obtained if these functions have a bell-shaped form with scalable variational parameters.

The $+-$ oddballs are not reported in this paper since they likely require the use of three body forces [25]. The ground state of the three-gluon system is calculated in our approach to be a 0^{-+} glueball (because of color, an annihilation diagram in [26] cannot contribute in the odd-C sector), but this is heavier than the corresponding p -wave excitation in the two-body system. It is interesting to note also that these 0^{-+} glueballs appear in the spectrum as apparent η_c excitations which have been reexamined in recent B-meson factory experiments, leading to a revised $\eta_c(2S)$ mass. Finally we present our $(PC = --)$ spectrum in the figure . For comparison we also show the good agreement with lattice computations. In addition we give an existing constituent gluon model calculation [2] and another by us. The constituent model treatment of [27] does not report on higher spin states and we leave it out. One can immediately observe that the leading oddball Regge trajectory, in analogy with the pomeron, the odderon, passes by the 3^{--} trajectory (three gluons, all spins aligned) and not by the 1^{--} as erroneously assumed in the literature. We look forward to further lattice computations for higher spin states to confirm this point.

Also shown in the figure is the conventional ω Regge trajectory. As can be seen and was to be expected due to the smaller color factor, it has much larger slope but higher intercept. At $t = 0$ it will clearly dominate scattering amplitudes and the odderon should not be visible, as is the case in low $-t$ searches at DESY [28]. Although in our best variational model the odderon will not be observable (by the time the two trajectories intersect, $t = -2.3 \text{ GeV}^2$ and $J < -1$, a linear extrapolation from timelike t is no more valid), in our approach, we can not categorically preclude future searches, either at DESY or BNL, discovering an odderon signature. Such searches, however, should also be conducted at sufficiently higher $-t$ (greater than $.5 \text{ GeV}^2$). Nevertheless

we remain pessimistic about the prospects for observing the odderon especially since a recent extension by Kaidalov and Simonov of their approach to higher spin states concurs with our findings [29].

In concluding we note this is a period of timely opportunities for exciting phenomenology related to dynamical gluon mass generation and propagator suppression. For example, the recent CLEO measurement [30] of the 1^{+-} meson mass, almost degenerate with the 1^{++} , shows that the hyperfine interaction is quite short range. This has traditionally been a conundrum of the quark model. If a long range Coulomb form is supposed for the physical gluon exchange and $\Delta V(r) \propto -\delta(r)$ the hyperfine potential has to be treated in perturbation theory (or the wavefunction falls to the origin) Then smearing parameters have to be introduced whose meaning is unclear. In contrast, the modern, field-theory based formulation cleanly separates the Cornell potential from the transverse exchange. This can be suppressed, its range being of order the inverse gluon gap $1/E_{min}(k)$, and the hyperfine interaction does not need to be so large as to provide π - ρ splitting, since chiral symmetry accomplishes this [31] and the transverse interaction can be treated in perturbation theory.

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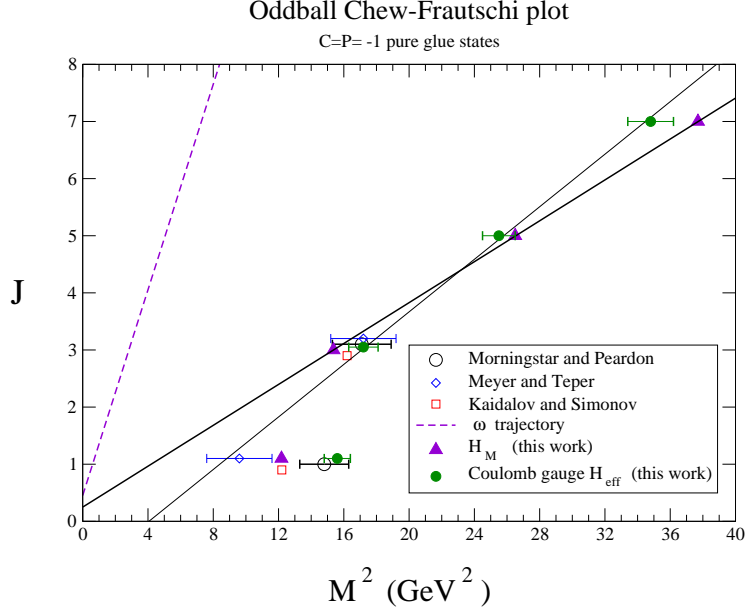


Figure 1: Chew-Frautschi plot for the J^{--} glueball spectrum. oddballs fall on linear Regge trajectories with a slope close to the pomeron, 0.25 in our preferred model H_{eff} . The leading trajectory starts at the 3^{--} oddball (three gluons with spins aligned, with excitations of increasing even orbital L) and has very low intercept at $J = 0$, subleading to the conventional ω Regge trajectory. For comparison we also include existing lattice and constituent gluon model estimates.

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